# NAG Toolbox for MATLAB

# f08ch

## 1 Purpose

f08ch computes an RQ factorization of a real $m$ by $n$ matrix $A$.

## 2 Syntax

```
[a, tau, info] = f08ch(a, 'm', m, 'n', n)
```

## 3 Description

f08ch forms the $RQ$ factorization of an arbitrary rectangular real $m$ by $n$ matrix. If $m \leq n$, the factorization is given by

$$A = \begin{pmatrix} 0 & R \end{pmatrix} Q,$$

where $R$ is an $m$ by $m$ lower triangular matrix and $Q$ is an $n$ by $n$ orthogonal matrix. If $m > n$ the factorization is given by

$$A = RQ,$$

where $R$ is an $m$ by $n$ upper trapezoidal matrix and $Q$ is again an $n$ by $n$ orthogonal matrix. In the case where $m < n$ the factorization can be expressed as

$$A = \begin{pmatrix} 0 & R \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = RQ_2,$$

where $Q_1$ consists of the first $(n - m)$ rows of $Q$ and $Q_2$ the remaining $m$ rows.

The matrix $Q$ is not formed explicitly, but is represented as a product of $\min(m, n)$ elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with $Q$ in this representation (see Section 8).

## 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The $m$ by $n$ matrix $A$.

### 5.2 Optional Input Parameters

1: **m – int32 scalar**

*Default*: The first dimension of the array **a**.

$m$, the number of rows of the matrix $A$.

*Constraint*: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

*Default*: The second dimension of the array **a**.

$n$, the number of columns of the matrix $A$.

*Constraint*: $\mathbf{n} \geq 0$.

### 5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

### 5.4 Output Parameters

1: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $m \leq n$, the upper triangle of the subarray $\mathbf{a}(1 : m, n - m + 1 : n)$ contains the $m$ by $m$ upper triangular matrix $R$.

If $m \geq n$, the elements on and above the $(m - n)$th subdiagonal contain the $m$ by $n$ upper trapezoidal matrix $R$; the remaining elements, with the array **tau**, represent the orthogonal matrix $Q$ as a product of $\min(m, n)$ elementary reflectors (see Section 3.2.6 in the F08 Chapter Introduction).

2: **tau(∗) – double array**

**Note**: the dimension of the array **tau** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The scalar factors of the elementary reflectors.

3: **info – int32 scalar**

**info** $= 0$ unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**info** $= -i$

If **info** $= -i$, parameter $i$ had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **tau**, 6: **work**, 7: **lwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

## 7 Accuracy

The computed factorization is the exact factorization of a nearby matrix $A + E$, where

$$\|E\|_2 = O\epsilon\|A\|_2$$

and $\epsilon$ is the ***machine precision***.

## 8 Further Comments

The total number of floating point operations is approximately $\frac{2}{3}m^2(3n-m)$ if $m \leq n$, or $\frac{2}{3}n^2(3m-n)$ if $m > n$.

To form the orthogonal matrix $Q$ f08ch may be followed by a call to f08cj:

```
[a, info] = f08cj(a, tau);
```

but note that the first dimension of the array **a** must be at least **n**, which may be larger than was required by f08ch. When $m \leq n$, it is often only the first $m$ rows of $Q$ that are required and they may be formed by the call:

```
[a, info] = f08cj(a(1:m,1:n), tau);
```

To apply $Q$ to an arbitrary real rectangular matrix $C$, f08ch may be followed by a call to f08ck. For example:

```
[a, c, info] = f08ck('Left','Transpose', a, tau, c);
```

forms $C = Q^{\mathrm{T}}C$, where $C$ is $n$ by $p$.

The complex analogue of this function is f08cv.

## 9 Example

```
a = [-5.42, 3.28, -3.68, 0.27, 2.06, 0.46;
     -1.65, -3.4, -3.2, -1.03, -4.06, -0.01;
     -0.37, 2.35, 1.9, 4.31, -1.76, 1.13;
     -3.15, -0.11, 1.99, -2.7, 0.26, 4.5];
[aOut, tau, info] = f08ch(a)
```

```
aOut =
    0.4338   -0.2576    7.4196    0.1856    0.2488   -1.7487
    0.1844    0.4800    0.4389    6.2025   -1.8915   -0.1372
    0.0199   -0.3168   -0.2365   -0.6046    5.6648    0.3604
   -0.2879   -0.0101    0.1819   -0.2468    0.0238   -6.4422
tau =
    1.5942
    1.3727
    1.3139
    1.6985
info =
           0
```